LITERATURE CITED

1. S. M. Kapustyanskii, "Propagation and reflection of three-dimensional plastic waves," Izv. Akad. Nauk SSSR Mekh. Tverd. Tela, No. 1 (1973).
2. G. V. Rykov and A. M. Skobeev, Measurement of Stresses in Soils Subjected to Momentary Loads [in Russian], Nauka, Moscow (1978).
3. N. V. Zvolinskii and G. V. Rykov, "Reflection of a plastic wave from a barrier," Prikl. Mat. Mekh., 27, No. 1 (1963).
4. A. N. Kovshov and A. M. Skobeev, "Reflection of a plastic wave striking a rigid wall at an angle," Izv. Akad. Nauk SSSR Mekh. Tverd. Tela, No. 1 (1973).
5. V. Z. Vlasov, General Theory of Shells, Gostekhizdat, Moscow-Leningrad (1949).
6. R. G. Yakupov, "Proportionality factor in problems of the interaction of a cylindrical shell with the surrounding elastic medium," Interaction of Shells with Fluid: Transactions of a Seminar. Kazan. Fiz.-Tekh. Inst. Akad. Nauk SSSR, Kazan (1981), Vol. 15.

THEORY OF INELASTIC STRAIN BASED ON NONEQUILIBRIUM OF THE STATE OF THE MATERIAL

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As any process which occurs at a finite rate, the inelastic strain of solids is always a thermodynamically nonequilibrium process. The transition from the given state to the equilibrium state is completed by stress relaxation, which converts elastic strain to inelastic strain. The theory of the deformation of solids has been constructed within this framework.

The basis of the classical theory of plasticity – the formation of inelastic strain when the given process is occurring in the equilibrium state – is only a convenient hypothesis [1, 2]. It leads to results which agree only with those experiments in which the rate of change in the external parameters is not greater than the rate of transition of the system (specimen) from the nonequilibrium state to the thermodynamically equilibrium state. In the theory of plasticity, such processes are referred to as quasistatic processes. In fact, fixing the external parameters in these processes means simultaneously fixing the parameters throughout the system as a whole. Indeed, nonequilibrium also exists during the process of plastic deformation, but the transition from this to the equilibrium state occurs only with a change in the external parameters – and is not seen after the latter become fixed. If the rate of change in these parameters is greater than the rate of transition of the system from the nonequilibrium state to the equilibrium state, then the nonequilibrium remains even after the external parameters stop changing. Thus, if they are fixed and subsequently kept constant, then the transition from the nonequilibrium state to the equilibrium state and associated phenomena, such as the formation of plastic strain, will continue until the establishment of thermodynamic equilibrium. The study of these phenomena was taken up in [3, 4].

If nonequilibrium during plastic deformation is not taken into account (and the state is assumed to be an equilibrium state), then, in accordance with the principle of the existence of a ground state, such deformation is a reversible process [5-7], i.e. the laws of thermodynamics are violated. Thus, the process cannot occur in nature. It follows from this that the theory of plasticity is only a model representation of the phenomenon of plastic inelastic deformation. It describes it as a process which occurs under certain conditions. Neither the theoretical or empirical accuracy requirements are high and the the existence of nonequilibrium is ignored. In fact, any inelastic deformation, including plastic deformation, is a nonequilibrium process. It is on this basis that the theory of plastic deformation has been constructed.

<u>1. Equilibrium and Nonequilibrium Stresses.</u> Proceeding as in the nonequilibrium thermodynamics of solids and basing our theory on the main conservation laws and the principles of objectivity, continuity, locality, and the existence of a ground state, we make the transition from an actual solid to a continuum and, within this continuum, we make

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another transition from global parameters of volume and pressure to local parameters – the tensors of stress and strain at a point of the medium. Here, the state at a point of the continuum is determined by the state of a closed thermodynamic system. The latter system is a neighborhood of the given point which is small enough to characterize the state at the point and large enough to reflect the properties of the continuum. This "closed system," i.e. a system which exchanges energy but not mass with its environment, is referred to as a phenomenological element [2].

In accordance with the principle of the existence of a ground state, a phenomenological element is in a thermodynamically nonequilibrium state at each moment when it is being deformed at a finite rate. As was shown in [3, 4], this representation implies that the tensor of the stresses σ_{ij} at each moment of irreversible strain is the sum of two tensor components:

$$\sigma_{ij} = \varphi_{ij} + \psi_{ij},\ i, j = 1, 2, 3. \tag{1.1}$$

Here, φ_{ij} are components of the stress tensor in the equilibrium state corresponding to the given state; ψ_{ij} are components of the stress tensor characterizing the difference between the given state and its corresponding equilibrium state. Thus, $\psi_{ij} = \sigma_{ij} - \varphi_{ij}$. Following [3, 4], we will refer to the components of the tensor φ_{ij} as equilibrium components and will refer to the components of the tensor ψ_{ij} as nonequilibrium components.

We will make use of the information presented in [8] to elucidate the physical nature and properties of nonequilibrium stresses. In accordance with this information, the deformation of a polycrystalline element is determined not only by the magnitude and character of the mean bonding forces, but also by the properties and state of the submicroscopic structure of the crystals, of the element – which undergo elastic distortion during loading. Of all of the possible structural distortions, we will focus on those which exist in a state of nonequilibrium and relax spontaneously at a temperature below the recrystallization temperature. As in [8], we will refer to stresses that are responsible for local elastic distortions of the lattice as local peak stresses. Being microscopic stresses over a characteristic local region, they are capable of reaching high values. On the one hand, local peak stresses increase with an increase in the loading rate and thereby help create a "nonequilibrium state" in the element. On the other hand, by simultaneously relaxing, they facilitate the transition of the element to the equilibrium state. The difference between the levels of the local peak stresses and the equilibrium stresses is the driving force behind relaxation as the process which converts elastic distortions of the lattice into residual (irreversible) strain.

The following must be noted here. In [8], references to local distortions of the crystalline lattice and the resulting local peak stresses meant only those distortions whose relaxation resulted in the formation of additional residual strain. As was emphasized in [8], the nature of this strain is the same as in plastic deformation. In the present study, we maintain the view that all irreversible (residual) deformation, including plastic deformation, is the result of a transition from a nonequilibrium state to an equilibrium state, i.e. at the microscopic level, it is the result of relaxation of the distorted structure of the element.

It was stated in [9] that the main mechanism responsible for rapid (plastic) deformation and transient and steady creep of polycrystalline solids is shearing of the components of its crystalline grains. While agreeing with this, we add that any microscopic shear is possible only in the presence of a driving force. This force is the local gradient of stresses in the form of the local peak stresses. Thus, irreversible strain is also examined within the framework of the nonequilibrium thermodynamics of solids. The entire phenomenon of irreversible deformation is determined by the aggregate effect of two opposite but interrelated thermodynamic processs: the formation and development of a nonequilibrium state in the form of distortions of the crystalline lattice of the element; the transition from the nonequilibrium state to the equilibrium state as a result of the removal of these distortions by relaxation. One of several types of irreversible strain – plastic, viscoelastic, or viscous – is realized, depending on the conditions under which relaxation occurs during the deformation process.

Within the framework of the thermodynamic approach being proposed here, the nonequilibrium stresses are by definition the macroscopic measure of the local peak stresses, while the result of their relaxation is one or a combination of the above three types of irreversible strain.

We will formulate the law governing the relaxation of nonequilibrium stresses as the macroscopic measure of the local peak stresses. The law governing the relaxation of these stresses was presented in [8]. Let nonequilibrium stresses $\Delta\psi_{ij}(\tau)$ be formed in an element during the time $\Delta\tau$. We assume that during the time du ($u > \tau$) the stresses $\Delta\psi_{ij}$ decrease by an amount which is proportional to $\Delta\psi_{ij}$. The rate at which this process takes place is greater, the shorter the time interval $u - \tau$. Thus, $d(\Delta\psi_{ij}) = \Delta\psi_{ij}(u)K(u-\tau)du$ ($K(u-\tau)$ is the kernel, which decreases with an increase in $u - \tau$). The solution of the given equation is $\Delta\psi_{ij}(t) = \Delta\psi_{ij}(\tau)Q(t, \tau)$, where

$$Q(t,\tau) = \exp\left[-\int_{\tau}^{t} K(u-\tau)\,du\right], \tag{1.2}$$

from which

$$\psi_{ij}(t) = \int_{0}^{t} \frac{d\psi_{ij}(\tau)}{d\tau} Q(t,\tau)\,d\tau. \tag{1.3}$$

The kernel of this relation should be such that the right side of (1.3) expresses the main properties of the local peak stresses. For example, following [9] we can put

$$Q(t-\tau) = \exp\left[-k(t-\tau)\right]. \tag{1.4}$$

Since $d\psi_{ij}$ is determined by the increment $d\sigma_{ij}$, then

$$d\psi_{ij} = q_{ijmn}d\sigma_{mn}. \tag{1.5}$$

It was shown in [4] that the increment of the tensor of total strain (ε_{ij}) is expressed in terms of the tensor of the nonequilibrium stresses and its increment in the following manner:

$$d\varepsilon_{ij} = A_{ijmn}d\psi_{mn} + B_{ijmn}\psi_{mn}dt. \tag{1.6}$$

Inserting (1.5) and (1.4) into (1.3) and then inserting (1.3) into (1.6), we obtain the relation

$$d\varepsilon_{ij} = a_{ijmn}d\sigma_{mn} + g_{ijmn}\int_{0}^{t} \frac{d\sigma_{mn}(\tau)}{d\tau} \exp\left[-k(t-\tau)\right]d\tau\,dt.$$

This is the fundamental relation in the strain theory that we are constructing on the basis of nonequilibrium representations.

2. Case of an Isotropic Solid. We will write differential tensor equation (1.6) for an isotropic material. Here, we require that its coefficients A_{ijmn} and B_{ijmn} take the same value in each coordinate system. We satisfy this requirement by taking A_{ijmn} and B_{ijmn} in the form of general isotropic tensors of rank four. In this case, we put

$$A_{ijmn} = l\delta_{ij}\delta_{mn} + a_1\delta_{im}\delta_{jn} + a_2\delta_{in}\delta_{jm},$$
$$B_{ijmn} = r\delta_{ij}\delta_{mn} + b_1\delta_{im}\delta_{jn} + b_2\delta_{in}\delta_{jm}$$

(l, r, a_1, a_2, b_1, b_2 are material constants). Inserting these values of A_{ijmn} and B_{ijmn} into (1.6) and taking into account the symmetry of the tensors of the nonequilibrium stresses ($\psi_{ij} = \psi_{ji}$), we finally obtain

$$d\varepsilon_{ij} = l\delta_{ij}d\psi + ad\psi_{ij} + (r\delta_{ij}\psi + b\psi_{ij})dt,$$
$$a = a_1 + a_2,\ b = b_1 + b_2,\ \psi = (1/3)\psi_{ii}. \tag{2.1}$$

For an isotropic material, Eq. (1.5) has the form

$$d\psi_{ij} = qd\sigma_{ij}. \tag{2.2}$$

Having inserted (2.2) into (1.3) and allowing for (1.4), we obtain

$$\psi_{ij}(t) = q\int_0^t \frac{d\sigma_{ij}(\tau)}{d\tau}\exp[-k(t-\tau)]\,d\tau. \tag{2.3}$$

Here, we write Eq. (2.1) as

$$\begin{gathered} d\varepsilon_{ij} = A\delta_{ij}d\sigma + Bd\sigma_{ij} + c\delta_{ij}\int_0^t \frac{d\sigma(\tau)}{d\tau}\exp[-k(t-\tau)]\,d\tau\,dt + \\ + g\int_0^t \frac{d\sigma_{ij}(\tau)}{d\tau}\exp[-k(t-\tau)]\,d\tau\,dt, \quad \sigma = \frac{1}{3}\sigma_{ii}. \end{gathered} \tag{2.4}$$

We determine the constants A and B in (2.4) with the strain increments by considering that, within the framework of the theory being constructed here, the only equilibrium strain is the reversible strain, i.e., the elastic strain found in accordance with Hooke's law. Thus,

$$\begin{gathered} d\varepsilon_{ij} = \frac{1+\nu}{E}\left(d\sigma_{ij} - \frac{3\nu}{1+\nu}\delta_{ij}d\sigma\right) + \left[g\int_0^t \frac{d\sigma_{ij}(\tau)}{d\tau}\exp[-k(t-\tau)]\,d\tau + \right. \\ \left. + c\delta_{ij}\int_0^t \frac{d\sigma(\tau)}{d\tau}\exp[-k(t-\tau)]\,d\tau\right]dt, \end{gathered} \tag{2.5}$$

where λ and μ are the Lamé constants; E is the Young's modulus; ν is the Poisson's ratio; g, c, and k are constant coefficients. The terms in square brackets in the right side of Eq. (2.5) determine the inelastic deformation of the isotropic material ($\varepsilon^{in}_{ij}(t)$) over time:

$$\begin{gathered} \dot{\varepsilon}^{in}_{ij}(t) = g\int_0^t \frac{d\sigma_{ij}(\tau)}{d\tau}\exp[-k(t-\tau)]\,d\tau + \\ + c\delta_{ij}\int_0^t \frac{d\sigma(\tau)}{d\tau}\exp[-k(t-\tau)]\,d\tau. \end{gathered} \tag{2.6}$$

Here, the second term shows that the theory being constructed here describes the inelastic deformation which develops with changes in hydrostatic pressure.

3. Determination of the Constants. The coefficients g, k, and c in Eq. (2.6) are material constants and are found from the following experiment. A specimen in the form of a thin-walled cylindrical tube is placed in tension at a constant rate. The tensile force is fixed beyond the yield point at a certain moment and is henceforth kept constant.

For uniaxial loading, Eq. (2.6) takes the form

$$\dot{\varepsilon}^{in}_z = R\int_0^t \frac{d\sigma_z(\tau)}{d\tau}\exp[-k(t-\tau)]\,d\tau, \quad R = g + \frac{1}{3}c \tag{3.1}$$

or, after integration by parts,

$$\dot{\varepsilon}^{in}_z = R\left[\sigma_z(t) - k\int_0^t \sigma_z(\tau)\exp[-k(t-\tau)]\,d\tau\right]. \tag{3.2}$$

We will describe an experiment involving the uniaxial tension of a specimen with subsequent holding under a constant stress. First let the specimen be subjected to tension at a constant rate $\dot{\sigma}$. Here, in accordance with (3.1)

$$\varepsilon_z^{\text{in}}(t) = \frac{R}{k}\dot{\sigma}\left[t - \frac{1}{k}(1 - \exp(-kt))\right].$$

Upon attainment of the value $\sigma_z = \sigma_1$, for $t = t_1$ the stress becomes equal to $\sigma_1 = \sigma_0 + \dot{\sigma}t_1$, while the inelastic strain

$$\varepsilon_1^{\text{in}} = \frac{R}{k}\dot{\sigma}\left[t_1 - \frac{1}{k}(1 - \exp(-kt_1))\right]. \tag{3.3}$$

We will fix the stress σ_1 and keep it constant. Then, in accordance with (3.2), the inelastic strain increases by the amount

$$\Delta\varepsilon_1^{\text{in}}(t) = \frac{R}{k}\sigma_1[1 - \exp(-kt)], \quad 0 \leqslant t \leqslant \infty,$$

the maximum value of which at $t \to \infty$ is

$$\Delta\varepsilon_{\max}^{\text{in}} = R\sigma_1/k. \tag{3.4}$$

The results found here are adequate to determine the constants R and k. Assuming that $\varepsilon_1^{\text{in}}$, $\Delta\varepsilon_{\max}^{\text{in}}$ and t_1 are known from an experiment, we find from (3.4) that $R/k = \Delta\varepsilon_{\max}^{\text{in}}/\sigma_1$. Inserting this value into (3.3), we obtain an equation to find the coefficient k:

$$\frac{\dot{\sigma}\Delta\varepsilon_{\max}^{\text{in}}t_1 - \varepsilon_1^{\text{in}}\sigma_1}{\dot{\sigma}\Delta\varepsilon_{\max}^{\text{in}}}k + \exp(-kt_1) = 1.$$

Thus, the theory of nonequilibrium strain developed here on the basis of thermodynamic representations of nonequilibrium during the inelastic deformation of solids is consistent with known physical laws. It represents elastic strain as the result of the transition of a system from a nonequilibrium to an equilibrium state. As the system makes this transition, it approaches the boundary between plastic and viscous strain that was introduced artifically for expediency in performing theoretical calculations. This boundary does not actually exist in physical processes.

LITERATURE CITED

1. L. I. Sedov, Continuum Mechanics, Vol. 2, Nauka, Moscow (1973).
2. A. A. Vakulenko, "Possibility of irreversible quasistatic processes in a macroscopic system," Prikl. Mat. Mekh., 48, No. 4 (1984).
3. K. N. Rusinko and É. I. Blinov, "Analytical description of the thermoelastoplastic deformation of solids," Prikl. Mekh., 11, No. 17 (1981).
4. É. I. Blinov, "Possible method of constructing a kinetic theory of deformation," Izv. Akad. Nauk SSSR Mekh. Tverd. Tela, No. 5 (1989).
5. M. A. Leontovich, Introduction to Thermodynamics. Statistical Physics [in Russian], Nauka, Moscow (1983).
6. "Reversible Process," Encyclopedic Dictionary of Physics [in Russian], Sov. Éntsiklopediya, Moscow (1983).
7. V. D. Sivukhin, "Editor's Notes," M. A. Leontovich. Introduction to Thermodynamics. Statistical Physics [in Russian], Nauka, Moscow (1983).
8. K. N. Rusinko, Theory of Plasticity and Transient Creep [in Russian], Vishcha Shkola, L'vov (1981).
9. K. N. Rusinko, Features of the Inelastic Deformation of Solids [in Russian], Vishcha Shkola, L'vov (1986).